2019 version

Faculty Member: Jonathan Mitchell **Faculty ID:** 10115122

Department: Mathematics and Statistics **Faculty Email:** mitchelljonat@sfasu.edu

Have you previously received a SURE award? No **If yes, when?**_____

If yes, how did you disseminate results from previous awards (successfully published a paper or obtained a grant, submitted paper or grant based on results, presented results at external conference, etc.):_____

Student's Name: Lee Henslee **Student ID:** 20034854

Major: Engineering Physics **Student Email:** hensleelw@jacks.sfasu.edu

Have you previously received a SURE award? No **If yes, when?**_____

Proposed SURE Project

Title of proposed SURE project: Mathematical Methods for Approximating the Van der Pol Equation

Description of proposed project (describe the scope of the project, including specific objectives):

Many of the scientific studies concerning phenomena that appear to be repetitive in nature (lasers, epidemics, etc.) involve systems of differential equations. The solutions to these equations are periodic and are thus referred to as oscillators. This project will investigate a nonlinear oscillator (e.g. Van der Pol, Rayleigh, Duffing) and the methods o approximating its solutions. A common feature of many nonlinear 2^{nd}-order differential equations is the difficulty or in some cases impossibility of stating an exact solution. The usual way around this dilemma for scientists and engineers over many decades has been to rely on sufficiently accurate partial sums of a convergent series of base functions. The base functions can be polynomials (power series), trig functions (Fourier series), or some other elementary function (Homotopy Analysis Method). The scope of this project will begin with readings and exercises related to series-expansions of solutions to nonlinear problems. It would then reach as far as material common to many graduate level Perturbation Methods courses. The primary objectives of this project include:

- Demonstrate a basic understanding of power-series expansions as solutions to differential equations by solving select exercises.
- Effectively utilize the numerical computing tool, MATLAB, in order to verify and display results of analysis.
- Determine the amplitude and phase for a weakly-damped nonlinear oscillator using the method of multiple scales.
- Determine the frequency of a conservative nonlinear system as a power series using the Poincare-Lindstedt method.
- Prepare a poster and presentation of the project with intentions to present in a suitable academic setting (such as the Texas Undergraduate Mathematics Conference in the Fall 2019).

In my section of MTH 333, Calculus III, Lee demonstrated that he is ready, able, and motivated to go beyond the classroom and succeed in a research project of this caliber. He often stayed after class to ask for clarification on key steps to exercises or even to pose the "follow-up" questions, which tended to go beyond the scope of the course. Lee will be in my section of MTH 337, Differential Equations, this Spring 2019. This will lay the ground work for this well-timed research project before the upcoming summer. However, there will be some assigned reading for Lee during the first summer session to supplement his preparation for this project.

Is this a new project or a continuation of a current project? If a continuation, what new work will be done as part of SURE?

The faculty member involved has researched methods of approximating solutions to nonlinear oscillators and published a few papers involving the topic. The most recent, in November 2017, investigated some of these methods applied to the Lienard Equation with cubic nonlinearity. The faculty member's experience here will help as he guides Lee through a few known but advanced results.

Potential impact or significance of research:

There is an extremely high potential for impact should this project be granted. Nonlinear oscillators are widely used in the sciences as applications in biological populations (epidemiology), laser dynamics, electrical engineering, drilling processes and more. The methods for approximating their solutions has been a growing field of mathematica analysis for over a century. In order to understand some of the more modern techniques like the Homotopy Analysis Method (2004), one must build a firm understanding of some of the more traditional methods such as the Poincare-Lindstedt method (formalized in 1959 & 1963). The proposed project is certainly substantial enough to present at the Texas Undergraduate Research Mathematics conference in the Fall semester 2019. In fact, the project may even be strong enough to be written up and submitted to an undergraduate research journal such as SIURO (SIAM Undergraduate Research Online) or the Minnesota Journal of Undergraduate Mathematics.

Research Design (approach/methodology):

In order to develop a solid base for some of the more advanced methods, Lee will review a few topics introduced in MTH 337 (he is enrolled in Spring 2019) during the month of June, 2019. These topics for review will include but are not limited to the forced harmonic oscillator, linear stability of systems of differential equations, and Euler's formula. The faculty member will then discuss with Lee how we use Euler's formula and its consequences to express the oscillator's complex amplitude and frequency and analyze its behaviors. There will be assigned readings and relevant exercises in order to reinforce these fundamentals. Along the way, the faculty mentor will show Lee how to confirm the results of his analysis with numerical approximations (computer simulation) using MATLAB. As material progresses and becomes more complicated, there will be a pattern of teaching (introductory discussion), individual work/study, and clarification (e.g. Q & A). This could include as many as 3-5 meetings per week as needed in order for the mentor to monitor Lee's progress and help the project move forward without doing the work for him. Once Lee has used power series to approximate a few linear differential equations (Ch 5 Adv. Engineering Math.), he will have joined two important ideas: oscillators as 2^{nd}-order differential equations and their solutions as a convergent series. The faculty member will then show how this concept extends to the proper use of the method of multiple scales, followed by the Poincare-Lindstedt method. With some guided help from the faculty mentor, both of these methods can be practiced and demonstrated with various exercises, which will be selected from Holmes' Perturbation Methods book. Motivated by the semi-conductor laser that uses the Van der Pol oscillator (Foutse et al. 2015), Lee will approximate its solutions using the Poincare-Lindstedt method and synthesize his results with what was published in the paper itself. Lee will then prepare a poster and presentation in order to disseminate the important results of this research project.

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Project timeline (activity/task and time to complete):

	Monday - Friday
June	Review the forced harmonic oscillator; both resonant and non-resonant cases. Read selected sections from Chapters 3 and 11 from Advanced Engineering Mathematics as well as Part 1: sections 2 - 14 from Mathematical Models by Haberman. During this time, I will be available for meetings if necessary.
15-Jul-19	Investigate the connections between real and complex forms of frequency and amplitude of solutions using Euler's formula. Use power series to solve a few linear differential equations (Ch 5 Adv Eng. Math). Develop MATLAB code to numerically solve and display solutions to several traditional nonlinear oscillators (Van der Pol, Duffing, Rayleigh, etc).
22-Jul-19	Investigate the Method of Multiples Scales. Read sections 3.1 - 3.2 from Holmes book, Introduction to Perturbation Methods. Write up solutions to exercises 1 (a) - (d) involving the method of multiple scales.
29-Jul-19	Read introduction and first section of Simplified Lienard Equation by HAM, by Mitchell (2017). Confirm results (at least the first correction term) using the Poincare Lindstedt method. Read the semiconductor laser paper by Foutse et al.
5-Aug-19	Apply the Poincare-Lindstedt method to approximate amplitude and frequency for the dimensionless Van der Pol oscillator. Compare results with those from the laser paper
12-Aug-19	Design and create a poster showing the main results of the project.

Literature review for project:

- Haberman, Richard. *Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow.* SIAM. 1998
- Zill, Dennis G. and Warren S. Wright. *Advanced Engineering Mathematics.* Jones & Bartlett. 5th edition. 2014.
- Strogatz, Steven H. *Nonlinear Dynamics and Chaos: with applications to physics, biology, chemistry, and engineering.* Westview Press. 2nd edition. 2015
- Holmes, M. H. *Introduction to Perturbation Methods.* Springer. 1995.
- Mitchell, Jonathan. *Simplified Lienard Equation by Homotopy Analysis Metho*d. Springer: Differential Equations and Dynamical Systems. Nov 2017. DOI 10.1007/s12591-017-0404-4
- Amore, Paolo, John P Boyd, and Francisco M Fernández. *High Order Analysis of the Limit Cycle of the Van Der Pol Oscillator.* 28 Nov, 2017. Cornell University. Subject: Mathematics, Numerical Analysis
- Foutse, Momo, S. T. Kingni, B. Nana, and P. Woafo. *Edge-emitting semiconductor laser driven by a van der Pol oscillator: analytical and numerical analysis.* Optical and Quantum Electronics. March 2015. 47: 3, pp 705-720.

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Description of research and professional skills that the student will develop from the project:

Lee is an Engineering Physics student planning to graduate in 2020. Although he is willing to enter the workforce right away in the mechanical engineering industry, he is also considering the idea of continuing his education within some graduate program. Either way, this project suits Lee very well. As an engineer, methods of approximating nonlinear problems are extremely useful primarily because most problems they face are nonlinear. As a possible future graduate student, the practice of independent study and research bode well for his training and success in a STEM discipline. In addition, the faculty member will help Lee develop and improve his computing ability by comparing his analytical results to those obtained numerically. That is, Lee will properly use computer code for MATLAB to numerically approximate the solution to any exercise or mathematical model he encounters during this research.

Description of the involvement and activities that the student and mentoring faculty will have in this project:

The student will meet with the faculty mentor in person at least 3-5 times a week. This allows the faculty member to introduce new concepts as well as monitor the student's research progress. The nature of the project allows a fair amount of remote communication via email, the exchange of MATLAB files, etc. Following the research project, the student will look to present the results of his research at the Texas Undergraduate Mathematics Conference and perhaps the SFA Undergraduate Research Conference.

Description of how you will disseminate results from the project:

Lee will present the results of this research project at a suitable academic conference for undergraduates. This could include the Texas Undergraduate Mathematics Conference in during the fall semester of 2019. Lee will design a poster with hopes to present at a conference or display in the Cole STEM building. In addition, any significant results will be written as an academic article and submitted to an appropriate undergraduate research journal.

Budget ($0-$500 with justification):

No materials will be needed.

Amount requested for supplies from SURE: $ 0

Amount requested for supplies from department: $ 0

Amount department will fund faculty stipend: $ 0

Amount department will fund student stipend: $ 500 -

Chair approval: Lisa Bueno

Chair Signature

I have reviewed and agreed to fulfill the expectations of the SURE award.

Student Signature

Faculty Signature

For internal purposes only:

Proposal Awarded _____ Proposal not award_____ Amount awarded: _____

Accounts to be used for award:_____